Dominik KUJAWA
UNIWERSYTET ZIELONOGÓRSKI, INSTYTUT STEROWANIA I SYSTEMÓW INFORMATYCZNYCH

Comparison of Four Subspace Identification Algorithms

mgr inż. Dominik KUJAWA

Autor jest doktorantem na Wydziale Elektrotechniki, Informatyki i Telekomunikacji Uniwersytetu Zielonogórskiego. W swojej pracy badawczej zajmuje się teorią sterowania, identyfikacją systemów dynamicznych metodami podprzestrzeni, systemami powtarzalnymi.

e-mail: administrtator@csd.com.pl

Abstract

In this paper, four subspace identification algorithms are compared and evaluated. The input-output data are acquired from a real industrial system of a soft drink manufacture. For this particular case of system identification, four algorithms, i.e. CVA, N4SID, MOESP, and SUBIDB3, are considered. Behavior of the models obtained by identification algorithms is compared with behavior of the system. Generally speaking, the constructions of stochastic realization theory are translated by subspace methods into procedures. The procedures which work on measured data are used for model building.
The name subspace identification is not used without a reason. These methods ought to be called subspace due to the fact that the basic objects are subspaces generated by the input-output data acquired from real systems being complicated mathematical systems. By going into more detailed explanation, we can clearly see that geometric operations are the necessary factor needed to compute estimates of the parameters.
Having taken into consideration the features of subspace methods, it turns out that it is not necessary to use canonical parameterizations. Thus, it is not essential and required to use iterative nonlinear optimization. Consequently, only non-complicated and numerically robust tools of numerical linear algebra, i.e. QR, SVD, and QSVD decompositions, are to be used. By coming to conclusion, we can state that a deeper system theoretic understanding of the used procedures, since the methods rely on the theoretical background of stochastic realization, is possible.
In this paper, the results of the four models identification gained by the subspace methods in innovation form

$$x(t+1) = Ax(t) + Bu(t) + Ke(t), \qquad x(0) = x0$$
$$y(t) = Cx(t) + Du(t) + e(t)$$

are shown. Namely, four algorithms implementation estimates models concur with the real system. The paper describes estimation of each model fit gained not only by the use of the three blocks but also the standard functions of subspace identification methods.

Streszczenie

W artykule sprawdzono i porównano 4 algorytmy identyfikacji metodami podprzestrzeni. Sygnały wejściowe i wyjściowe pozyskano z rzeczywistego systemu fabryki produkującej napoje. Dla tego szczególnego przypadku rozważono 4 algorytmy identyfikacji takie jak: CVA, N4SID, MOESP i SUBIB3. Zachowanie modeli otrzymanych w wyniku identyfikacji porównano z zachowaniem systemu. Ogólnie mówiąc, konstrukcje stochastycznej teorii realizacji są tłumaczone przez metody podprzestrzeni na procedury. Procedury te pracują na danych, które są używane do budowy modelu.
Nazwa identyfikacja metodami podprzestrzeni nie jest używana bez powodu. Metody te powinny nazywać się metodami podprzestrzeni z powodu faktu, że podstawowe obiekty są podprzestrzeniami wygenerowanymi z danych sygnałów wejściowo – wyjściowych pozyskanych ze skomplikowanych rzeczywistych systemów matematycznych. Przechodząc do bardziej szczegółowego omówienia można zauważyć, że pewne operacje geometryczne są niezbędne, aby obliczyć oceny parametrów.
Uwzględniwszy cechy metod podprzestrzeni, okazuje się, że nie jest konieczne stosowanie parametryzacji kanonicznej. Tak, więc nie jest wymagane stosowanie iteracyjnych metod optymalizacji nieliniowej W konsekwencji, jedynie nieskomplikowane i numerycznie odporne narzędzia numeryczne algebry liniowej takie jak rozkłady QR, SVD i QSVD są stosowane. Podsumowując, możemy stwierdzić, że głębsze teoretyczne zrozumienie użytych procedur jest możliwe, ponieważ metody te są oparte na teoretycznych podstawach realizacji stochastycznych.
W artykule tym przedstawiono, wyniki identyfikacji metodami podprzestrzeni czterech modeli w postaci innowacyjnej

$$x(t+T_s) = Ax(t) + Bu(t) + Ke(t), \qquad x(0) = x0$$
$$y(t) = Cx(t) + Du(t) + e(t)$$

Mianowicie, modele estymowane za pomocą czterech algorytmów są zbieżne z systemem rzeczywistym. W artykule przedstawiono oszacowania dopasowania każdego z modeli otrzymane nie tylko przy użyciu trzech bloków danych, ale także standardowych funkcji identyfikacji metodami podprzestrzeni.

Słowa kluczowe: modele w przestrzeni stanów, metody podprzestrzeni, algorytmy identyfikacji, estymacja parametrów.

Keywords: state space models, subspace methods, identification algorithms, parameter estimation.

1. Introduction

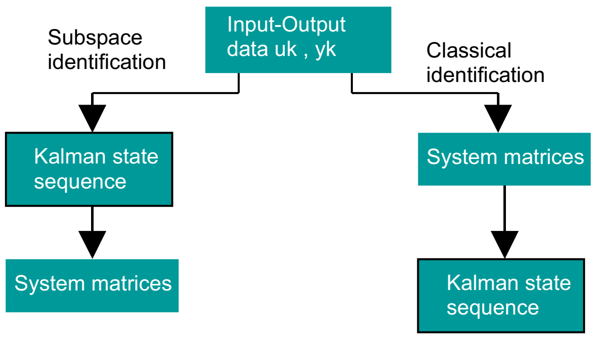
About twenty years ago identification algorithms using subspace methods were developed from these most popular ones, which are as follows:

Canonical variate analysis [11].

Numerical algorithm of subspace state-space system identification [7].

Multivariate output-error state space [5].

This is especially true for high-order multivariable systems, for which it is not trivial to find a useful parameterization among all parameterizations. This parameterization is needed to start up the classical identification algorithms [4], which means that a-priori knowledge of the system order and of the observability or (controllability) indices is required.



Rys. 1. Lewa strona rysunku ilustruje identyfikację metodą podprzestrzeni SUBIDB3: najpierw wyznaczana jest sekwencja stanów systemu, a następnie obliczane są macierze systemu. Po prawej stronie pokazane jest podejście klasyczne: najpierw wyznacza się macierze systemu, a następnie oblicza się sekwencję stanów systemu.

Fig. 1. The left hand side shows the SUBIDB3 approach: first the (Kalman) states, then the system matrices. The right hand side is the classical approach: first the system, and then an estimate of the state.

In each of the four algorithms, most of this a-priori parameterization can be avoided. Only the system order is needed and it can be determined through inspection of the dominant singular values of a matrix that is calculated during the identification. The state space matrices are not calculated in their canonical forms (with a minimal number of parameters), but as full state space matrices in a certain, almost optimally conditioned basis (this basis is uniquely determined, so that there is no problem of identifiability). This implies that the observability (or controllability) indices do not have to be known in advance.

Other major advantages are that the four algorithms are non-iterative, with no non-linear optimization part involved. This is why they do not suffer from the typical disadvantages of iterative algorithms such as no guaranteed convergence, local minima of the objective criterion and sensitivity to initial estimates. For classical identification, an extra parameterization of the initial state is needed when estimating a state space system from data measured on a plant with a non-zero initial condition. Final advantages of the four algorithms are that they handle non-zero initial states without any extra effort. Fig. 1 shows how these subspace algorithms differ from classical identification schemes. The four algorithms tested in this paper identify a general state space model for deterministic systems directly from the input-output data [2], [3].

2. Problem Statement

The input signal $\{u(t)\}$ is a p-dimensional vector and the output signal $\{y(t)\}$ is a m-dimensional vector. The model contains coefficients (parameters) grouped into matrices with dimensions adjusted to the number of inputs and outputs of the system and the model is created in the state space in its innovation form.

$$\begin{aligned} x(t+T_s) &= Ax(t) + Bu(t) + Ke(t), \qquad x(0) = x0 \\ y(t) &= Cx(t) + Du(t) + e(t) \end{aligned} \tag{1}$$

where: $u(t) \in R^p$ and $y(t) \in R^m$ denote the observed input and output signals, respectively, $x(t) \in R^n$ is the state-vector and n is the system order. We assume that the system is minimal in the sense that the system cannot be described by a state-space model of order less than n. $e(t) \in R^m$ denotes the zero mean white innovation process [10].

The matrix $A \in R^{n \times n}$ is called the (dynamical) system matrix. It describes the dynamics of the system (as completely characterized by its eigenvalues).

The matrix $B \in R^{n \times p}$ is the input matrix, which represents the linear transformation by which the deterministic inputs influence the next state.

The matrix $C \in R^{m \times n}$ is the output matrix, which describes how the internal state is transferred to the outside world in the measurements y_k.

The matrix $D \in R^{m \times p}$ is called the direct feedthrough term. In continuous time systems this term is most often 0, which is not the case in discrete time systems due to the sampling [6].

The matrix $K \in R^{n \times m}$ is called Kalman filtr.

In the model (1), the matrices A, B, C, D and the covariance matrix of the innovations are unknown and need to be estimated from observations of $\{y(t)\}$ (m-dimensional) and $\{u(t)\}$ (p-dimensional) for $t = 1,2,...,N$. Typically, also the system order n is unknown and needs to be estimated [8], [9].

The model (1) generates the observed data: $\{y(t)\}$ and $\{u(t)\}$. Let $\{x(t)\}$ and $\{e(t)\}$ be the sample paths of the corresponding state and the innovation processes, respectively. Suppose (ideally) that we have observations on some (hopefully very long) time intervals $[0, N]$, of one sample path $\{y(t)\}$, $\{u(t)\}$, $\{x(t)\}$ of the processes [$\{y(t)\}$, $\{u(t)\}$, $\{x(t)\}$, $\{y(t)\}$, $\{u(t)\}$, $\{x(t)\}$] respectively. Since these processes generate the data, it is obvious that the finite `tail' matrices, Y_t, U_t, X_t, (2) constructed at each time moment t from the observed samples by the recipe

$$\begin{aligned} Y_t &:= [y(t)y(t+1)...y(t+N-1)], \\ U_t &:= [u(t)u(t+1)...u(t+N-1)], \\ X_t &:= [x(t)x(t+1)...x(t+N-1)], \end{aligned} \tag{2}$$

also satisfy (1), i.e.,

$$\begin{cases} X_{t+1} = AX_t + BU_t + KE_t, \\ Y_t = CX_t + DU_t + E_t, \end{cases} \tag{3}$$

where $E_t := [e(t)e(t+1)...e(t+N-1)]$, is the innovation tail. This equation can be interpreted as a regression model. It is straightforward to see that, if the tail matrices X_{t+1}, Y_t, U_t, X_t are given, then one can solve (3) for the unknown parameters (A, B, C, D), say by least squares. Hence we have an ideal situation, when we have an input, an output, and the corresponding state sequence at two successive time instants t and $t+1$ available, the identification of the parameters (A, B, C, D) of the system (1) is a rather trivial matter. In practice, X_{t+1} and X_t are not available and will have to be estimated from the input-output data. This is the crucial step in most subspace identification algorithms [1].

3. Modeling of the perform-to-bottle production cycle process

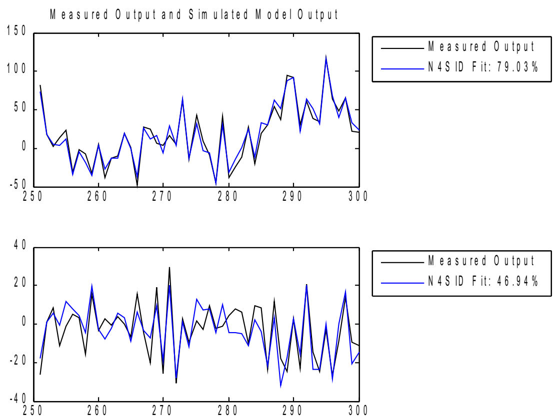
The first step is preparation of data for system identification using subspace methods. The goal here is testing and comparing of the obtained models. The data was acquired from a real industrial system of a Soft Drink manufactures, which is Classic Soft Drink. The data contains blow-molding machine input settings such as: u_1 – intensity, u_2 – rotational speed, u_3 – compressed air pressure for performs blowing. In a created object, the output signals y_1, y_2 –are the thickness of bottles sides and their diameter.

4. Identification results

In this section, we are comparing the responses of the models obtained using the four algorithms with the real system responses.

4.1 N4SID

For identification, the standard subspace algorithm N4SID, implemented in Matlab by default, was used, which worked with the same 250 samples in this parameter estimation as in another system identification algorithms. Figure 2 shows the comparison between model and the real system responses (50 samples used for comparison validation of the results).

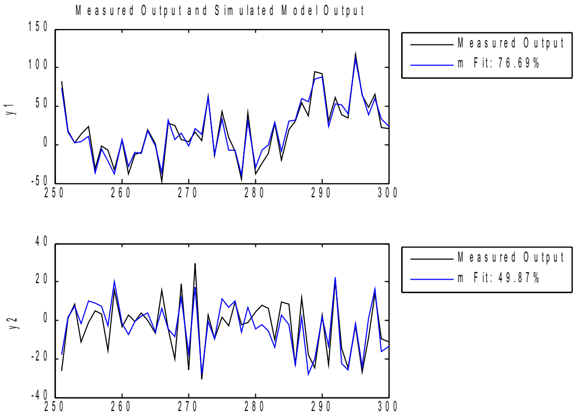


Rys. 2. Porównanie odpowiedzi systemu i modelu otrzymanego za pomocą algorytmu N4SID.

Fig. 2. Comparison between the responses of the real system and the model obtained using N4SID algorithm.

4.2 MOESP

For identification, the standard subspace algorithm MOESP, implemented in Matlab by default, was used, which worked with the same 250 samples in this parameter estimation as in the other system identification algorithms. Figure 3 shows the comparison between model and the real system responses (50 samples used for comparison validation of the results).

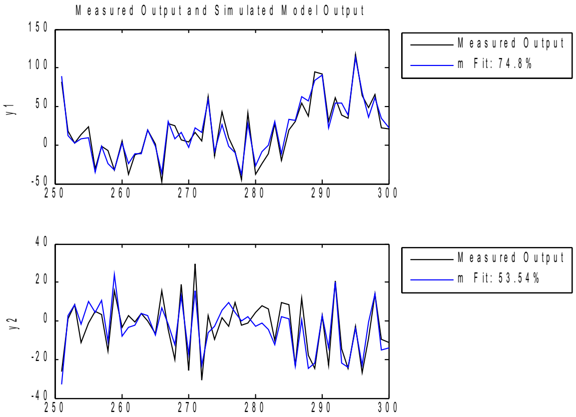


Rys. 3. Porównanie odpowiedzi systemu i modelu otrzymanego za pomocą algorytmu MOESP.

Fig. 3. Comparison between the responses of the real system and the model obtained using MOESP algorithm.

4.3 CVA

For identification, the standard subspace algorithm CVA, implemented in Matlab by default was used, which worked with the same 250 samples in this parameter estimation as in the other system identification algorithms. Figure 4 shows the comparison between the results of activity model and the real system (50 samples used for comparison validation of the results).

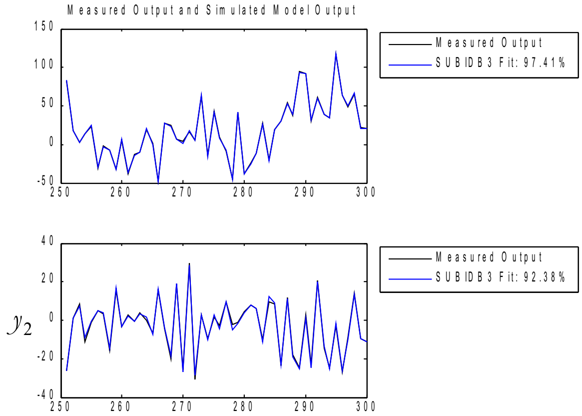


Rys. 4. Porównanie odpowiedzi systemu i modelu otrzymanego za pomocą algorytmu CVA.

Fig. 4. Comparison between the responses of the real system and the model obtained using CVA algorithm.

4.4 SUBIDB3

For comparison, the subspace identification algorithm SUBIDB3 was used in which a block of current signal values was added. The algorithm was implemented in Matlab. Fig. 5 shows the comparison between the response of the obtained model and the response of the real system.



Rys. 5. Porównanie odpowiedzi systemu i modelu otrzymanego za pomocą algorytmu SUBIDB3.

Fig. 5. Comparison between the responses of the real system and the model obtained using SUBIDB3 algorithm.

5. Conclusions

In this paper four subspace algorithms have been compared. Particular attention was paid to the new approach to subspace identification, which use three-block partitions of data matrices. Each of the three blocks can be seen as serving a different function, as an instrumental variable, a Kalman filter state observer, or a computational data set. A significant advantage of the new approach is that it gives unbiased estimates of the system matrices with the state sequence approach in a straightforward way. In the examined identification example, the N4SID, MOESP and CVA algorithms are faster than SUBIDB3. In computing, however, fitting the model output to real data is decidedly more accurate in SUBIDB3 case.

6. References

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Tytuł: Porównanie czterech algorytmów identyfikacji metodami podprzestrzeni

Artykuł recenzowany